Accurate Measurements with Image-Assisted Total Stations and Their Prerequisites

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Abstract: Video theodolites were used in the 1980s for highly accurate, automated measurements. However, they disappeared from the market and research on these instruments was done by only a few institutions using self-made prototypes. Because of the release of new-generation image-assisted total stations (IATS) by different manufacturers since 2004, these instruments have become relevant again for broader user groups. In this article, different error sources that occur when working with an IATS are assessed. The theoretical origins of these errors are discussed, their dependence on the measurement geometry is worked out, and strategies for avoidance and modeling are provided. In experimental geodetic network measurements, the impact of the different error sources on the results are evaluated. With standard deviations of a few 0.01 mm for the estimated three-dimensional (3D) coordinates, it is demonstrated that the telescope camera of a modern IATS can be used as a highly accurate measurement sensor. **DOI: 10.1061/(ASCE)SU.1943-5428.0000208.** © *2016 American Society of Civil Engineers*.

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Introduction

The integration of a camera into the telescope of a total station has become standard for high-end instruments from different manufacturers (Wagner et al. 2014; Leica 2015; Topcon 2016). These socalled image-assisted total stations (IATS) can be used in new fields of application. Examples are automated astrogeodetic measurements (Schirmer 1994) or automated, highly accurate geodetic network measurements without using retroreflective prisms (Guillaume et al. 2012). Further applications are the measurement of highly frequent oscillations (Hauth et al. 2013), optical plumbing (Knoblach 2011), rockfall monitoring (Reiterer et al. 2010), or the generation of textured three-dimensional (3D) models (Scherer 2007). Another promising application is the monitoring of civil engineering structures (Bürki et al. 2010; Wagner et al. 2013) in which an IATS can provide a fully contactless measurement system (Ehrhart and Lienhart 2015b).

Applications with high demand for accuracy require a detailed understanding of the used measurement equipment and its limitations. Therefore, possible error sources that occur when working with an IATS are investigated in this article. Three major issues, namely

- Imperfect modeling of the relationship between image coordinates and theodolite angles,
- Imperfect calibration of the camera, and
- Target-specific problems

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are addressed, and their impact on a typical surveying task (geodetic network measurement) is demonstrated.

It is shown that a correct modeling of the previous error sources and a sufficient warm-up time of the instrument allow automated angle measurements with an accuracy of 0.1 milligon (mgon) without using retroreflective prisms as targets. Accordingly, highly accurate measurements of selected 3D points with standard deviations of a few 0.01 mm are possible.

For the experimental measurements, a commercially available state-of-the-art IATS [Leica MS60 I R2000 (Leica Geosystems, Heerbrugg, Switzerland)] was used with specified standard deviations for manual horizontal and vertical angle measurements of 0.3 mgon (Leica 2015, p. 64). The used IATS is equipped with a telescope camera, also referred to as a *coaxial* camera, which is located in the optical path of telescope. Its image data therefore benefits from the $30 \times$ optical magnification of the telescope.

Relationship between Image Coordinates and Theodolite Angles

The theodolite angles of the IATS are given in the well-known polar coordinate system with horizontal H_z and vertical V angles. The image coordinates u and v are given on the image sensor plane π_{image} . In an ideal situation π_{image} is perpendicular to the optical axis of the telescope, and the origin of the image coordinate system is the principal point, i.e., the intersection of the optical axis with π_{image} . For image-based measurements a target is observed on the image sensor, and the image coordinates of the target need to be related to the theodolite angles of the IATS for further processing. An intuitive approach for obtaining the theodolite angles to the target would be

$$Hz_{T,*}[\operatorname{rad}] = Hz[\operatorname{rad}] + u[\operatorname{px}]/f[\operatorname{px}]$$
(1)

$$V_{T,*} [\operatorname{rad}] = V [\operatorname{rad}] + v [\operatorname{px}] / f [\operatorname{px}]$$
(2)

where f = focal length [expressed in pixels (px)]; and further calibration parameters are omitted.



Fig. 1. Geometry for relating image coordinates to theodolite angles (data from Schirmer 1994 and Juretzko 2004): (a) overview; (b) detail

However, it can be seen in Fig. 1 that the relationships of Eqs. (1) and (2) are insufficient. It is necessary to interpret the image coordinates of the target as the gnomonic projection of the respective theodolite angles to the image sensor plane.

Given the ideal image coordinates u and v and the focal length f of the camera (cf., the next section), the inverse gnomonic projection to compute the theodolite angles to the target reads (adapted from Snyder 1987, p. 167)

$$Hz_T = Hz + \arctan \frac{u \sin c}{\rho \cos c \sin V + v \sin c \cos V}$$
(3)

$$V_T = \arccos\left(\cos c \cos V - \frac{v \sin c \sin V}{\rho}\right) \tag{4}$$

where

$$\rho = \sqrt{u^2 + v^2} \tag{5}$$

$$c = \arctan \frac{\rho}{f}.$$
 (6)

For the used IATS with a 2,560 \times 1,920-px image sensor and a focal length of $f \approx 105,000$ px (cf., the next section), Eqs. (3) and (4) can be simplified to

$$Hz_T = Hz + \arctan \frac{u}{f \sin V + v \cos V} \tag{7}$$

$$V_T = \arccos\left[\left(1 - \frac{\rho^2}{2f^2}\right)\cos V - \frac{v\sin V}{f}\right]$$
(8)

retaining maximum deviations from the rigorous form given by Eqs. (3) and (4) of less than 0.1 mgon for all possible configurations of vertical angles and positions of the target on the image sensor plane.

Investigating Eqs. (7) and (8) reveals a dependency on the vertical angle V for both corrections. For Eq. (7) the effect of V is stronger due to the large value of f in the term f sin V. At an almost horizontal sighting with V = 95 gon, the insufficient relationship of Eq. (1) already produces deviations to Eq. (7) of up to 3 mgon. For V =90 gon, these deviations amount to 11 mgon for targets observed in the boundary areas of the image sensor. Bürki et al. (2010) therefore proposed a simplified relationship according to [adapted to the notation and using tan $(u/f) \approx u/f$]

$$Hz_{T,s} = Hz + \arctan\frac{u}{f\sin V} \tag{9}$$

$$V_{T,s} = V + \frac{v}{f}.$$
(10)

Eqs. (9) and (10) provide a convenient method for relating the image coordinates of a target to the respective theodolite angles. Therefore, the absolute mapping errors

$$\Delta = \sqrt{(Hz_T - Hz_{T,s})^2 + (V_T - V_{T,s})^2}$$
(11)

between Bürki et al. (2010) and the thorough computation given by Eqs. (7) and (8) are investigated. In Fig. 2 the boundary areas for target positions on the image sensor resulting in errors of less than 0.1 mgon are illustrated for different vertical angles. For increasing vertical angles from the horizon (V = 100 gon), smaller areas on the image sensor can be used for highly accurate measurements.

For example, at a vertical angle of 30 gon, targets within a distance of 85 px from the principal point of the image sensor retain mapping errors of less than 0.1 mgon when using the simplified mapping relationship of Eqs. (9) and (10). For the used IATS with a scale factor of 0.61 mgon/px (cf., the next section), 85 px corresponds to about 50 mgon. For applications requiring an accurate measurement to a single target for one telescope position, a deviation between the actual telescope position and the direction to the target of less than 50 mgon can easily be established by an iterative positioning of the telescope. However, when measuring angles to targets imaged at a large distance from the principal point, the thorough mapping relationship of Eqs. (7) and (8) should be used.



Fig. 2. Target positions retaining absolute mapping errors of less than 0.1 mgon for different vertical angles (gon) of the investigated IATS

Applications for the latter are the measurement of multiple targets at one telescope position or the calibration of the camera, which is discussed in the next section.

Tilt Correction

The relationship between image coordinates and theodolite angles as given by Eqs. (7) and (8) is not affected by a tilt of the IATS (Wasmeier 2009, p. 20). However, in a tilted setup the vertical axis of the IATS does not correspond to the zenith direction, and the theodolite angles to the target H_{Z_T} and V_T need to be corrected. Modern total stations comprise a two-axis tilt sensor that measures longitudinal *il* and transverse *it* inclinations relative to the horizontal pointing direction H_Z .

The tilt correction of the theodolite angles can be done by (Walser 2004, p. 56)

$$Hz_{T,\text{tilt}} = Hz_T + it_T \cot V_{T,\text{tilt}}$$
(12)

$$V_{T,\text{tilt}} = V_T + il_T \tag{13}$$

where the longitudinal il_T and transverse it_T inclinations relative to the direction to the target Hz_T are used. The computation of il_T and it_T from the measured il and it is given by Walser (2004, p. 56).

Camera Calibration

The image coordinates of a target on the image sensor plane are related to theodolite angles by Eqs. (7) and (8) given the corrected image coordinates u and v and the focal length f of the camera. To obtain these values, a calibration of the IATS' camera is necessary. The calibration of a modern IATS is a well-studied topic (Walser 2004; Wasmeier 2009; Knoblach 2011) in which the camera can either be calibrated separately or in combination with the axis errors of the total station. For applications requiring a high accuracy, two-face measurements are used to eliminate the axis errors of the total station. Accordingly, a calibration of the camera only is addressed in this article.

Following Schirmer (1994, p. 27), it is assumed that

- The image sensor plane is orthogonal to the optical axis of the telescope,
- The u- and v-axes on the image sensor are orthogonal, and
- The focal lengths are equal for the *u* and *v*-direction.



Fig. 3. Relationship between measured (u', v') and corrected (u, v) image coordinates

It is also possible to estimate parameters for the previous assumptions (Walser 2004, p. 49), which was not found to be necessary for the used IATS. Hence, the corrected image coordinates u and v can be calculated from the measured image coordinates u' and v' by

$$\begin{bmatrix} u \\ v \end{bmatrix} = \left(1 + K_1(r^2 - r_0^2)\right) \begin{bmatrix} \cos \kappa & -\sin \kappa \\ \sin \kappa & \cos \kappa \end{bmatrix} \begin{bmatrix} u' - u_0 \\ v' - v_0 \end{bmatrix}$$
(14)

where u_0 and v_0 = principal point of the image sensor; and κ = rotation angle of the image sensor about the optical axis of the telescope (Fig. 3). To correct for radial distortion, the factor K_1 is applied in dependence of the radial distance to the principal point

$$r = \sqrt{\left(u' - u_0\right)^2 + \left(v' - v_0\right)^2}.$$
(15)

The distortion polynomial is typically shaped with a second zero crossing at a radial distance of r_0 (Kraus 2007, p. 57). For the used IATS r_0 was fixed to 1/3 of the image diagonal (1,067 px) during the calibration process.

The principal point is the intersection of the optical axis of the telescope with the image sensor, i.e., the location of the telescope's crosshairs in the image, which is the same for both telescope faces. The coordinate systems of the measured image coordinates (Fig. 3) are equal for both faces in which the origin is in the top left corner, u' is counted from left to right, and v' is counted from top to bottom. Accordingly, the image coordinates (not the location on the image sensor) of the principal point differ between Faces I and II in which

$$u_{0,\rm II} = w - u_{0,\rm I} \tag{16}$$

$$v_{0,\rm II} = h - v_{0,\rm I} \tag{17}$$

where w and h = width and the height of the image sensor. For twoface measurements in which the face averages

$$\bar{u} = (u'_{\rm I} - u_{0,\rm I} + u'_{\rm II} - u_{0,\rm II})/2 = (u'_{\rm I} + u'_{\rm II} - w)/2$$
(18)

$$\bar{\nu} = (\nu'_{\rm I} - \nu_{0,\rm I} + \nu'_{\rm II} - \nu_{0,\rm II})/2 = (\nu'_{\rm I} + \nu'_{\rm II} - h)/2 \tag{19}$$

are computed, the coordinates of the principal point u_0 and v_0 cancel out.

To determine the three remaining calibration parameters f, κ , and K_1 , the total station's ability to rotate its telescope by precisely known angles is used (Huang and Harley 1989). For a stable setup of the IATS and a circular target, established in the temperature-controlled laboratory with a basement decoupled from the building, the target was imaged at different positions on the image sensor. To estimate the distortion coefficient K_1 of Eq. (14) reliably, target positions in the boundary area of the image sensor also were used. It is therefore necessary to use a thorough relationship between the image coordinates and the theodolite angles to avoid mapping errors (cf., the previous section).

Inserting Eq. (14) into Eqs. (7) and (8) establishes a relationship between the measured image coordinates u' and v'; the calibration parameters f, κ , and K_1 ; and the measured theodolite angles H_z and V. Because the calibration parameters are used to correct two-face measurements in later applications, the two-face averages of the image coordinates and the theodolite angles are also used in the calibration procedure.

The computation of the calibration parameters is done by a leastsquares adjustment according to the Gauss-Helmert model and will not be discussed further in this work. For the prototype of an IATS used by Knoblach (2011), the calibration parameters depend on the actual focus position of the telescope. This is also the case for the state-of-the-art IATS investigated in this paper. Similar to Knoblach (2011, p. 106), the most important calibration parameter f can be modeled as a polynomial of second degree of the focus position. The calibration was performed for seven representative focus positions and received average values for the calibration parameters of about f = 104,560 px, $\kappa = -19$ mgon, and $K_1 = -7.2/10^{10}$ px⁻². The average focal length corresponds to a scale factor of about 0.61 mgon/px. The standard deviations of the estimated calibration parameters, resulting from the least-squares adjustment, are similar for all investigated focus positions in which the values are about $s_f = 1$ px, $s_{\kappa} = 1$ mgon, and $s_{K1} = 0.2/10^{10}$ px⁻².

In Fig. 4 the residuals of a selected calibration according to the scanning approach of Huang and Harley (1989) are shown for different sets of estimated calibration parameters. Along with the focal length *f*, which is needed to relate the pixel coordinates to angles, it is evident that the rotation of the image sensor κ and the distortion coefficient K_1 also need to be estimated. This is emphasized by Table 1.

Without estimating κ [Fig. 4(a)], the residuals reflect the rotation of the image sensor about the optical axis of the telescope in which the residuals' magnitudes increase with increasing distance to the principal point. Without estimating K_1 [Fig. 4(b)], the residuals reflect the radial distortion of the image sensor in which the direction of the residuals is radial symmetric with respect to the principal point. The magnitude of the residuals is small in areas around a radial distance of $r_0 = 1,067$ px to the principal point, which represents the second zero crossing of the distortion polynomial from Eq. (14). When estimating f, κ , and K_1 [Fig. 4(c)], the residuals only include random measurement errors. Therefore, with the used IATS it is not necessary to estimate further parameters, e.g., different focal lengths for the *u*- and *v*-direction, as proposed by Walser (2004, p. 49).

From Figs. 4(a and b) and Eq. (14) it can be seen that the impact of the calibration parameters κ and K_1 on the corrected image



Fig. 4. Residuals (lines) at different target positions on the image sensor (dots) for a camera calibration at 6.3 m and different calibration parameters taken into account: (a) *f* and K_1 estimated (κ neglected); (b) *f* and κ estimated (K_1 neglected); (c) *f*, κ , and K_1 estimated

Table 1. Mean Value, Standard Deviation, and Maximum of the Residuals for Different Sets of Estimated Parameters

Estimated parameters	Mean (mgon)	Standard deviation (mgon)	Max (mgon)
f, K_1	0.18	0.08	0.37
<i>f</i> , к	0.17	0.10	0.60
f, κ, K_1	0.07	0.04	0.15

coordinates u and v becomes larger for increasing distances to the principal point. Also, an uncertainty of the estimated focal length f has a larger effect on large image coordinates because u and v occur in the numerators of Eqs. (7) and (8). This means that errors caused by an imperfect camera calibration can be kept small when observing targets that are imaged in the vicinity of the principal point.

Target-Specific Problems

Image-based measurements can be conducted with different targets in which the shape of the target projected to the image sensor is actually analyzed. Errors occur if the computed direction to the detected target (on the image sensor) does not coincide with the direction to the corresponding real-world object. The most frequently used target shapes involve circles and corners (discussed in the following subsections) in which both shapes are subject to the mentioned error.

Another method for computing the direction to a target is the template-matching approach. Here, a given template of the target is searched in the image containing the actual target. The target template can either result from a previous measurement or can be a synthetic pattern that is appropriate for the shape of the actual target (Walser 2004, p. 76ff.). In this article, the template-matching approach is not investigated further but Ehrhart and Lienhart (2015b) are mentioned for a discussion of possible errors linked with this approach.

Ellipse Center Offset

In many applications image-based systems are used in combination with circular targets. When mapping these targets on an image sensor the circles appear as ellipses under the associated perspective projection (Hartley and Zisserman 2004, p. 37; Davies 2012, p. 460). The center of the ellipse on the image sensor can be efficiently computed by means of a least-squares adjustment (Fitzgibbon et al. 1996) based on the contour of the ellipse. However, the center of the detected ellipse does not coincide with the center of the circular target if the target plane is not parallel to the image sensor plane (Dold 1996; Ahn et al. 1999; Luhmann 2014).

To find the center of the circular target in the image, the detected ellipse center needs to be shifted toward the vanishing line (Hartley and Zisserman 2004, p. 217) of the target plane by

$$\epsilon = \frac{b^2}{d} \tag{20}$$

where b = semiminor axis of the ellipse; and d = distance (in image coordinates) of the ellipse center to the vanishing line (Davies 2012, p. 460). For evaluating Eq. (20), b results from the least-squares adjustment of the ellipse, where d and the direction of the correction ϵ are subject to further computations.

The vanishing line of the target plane π_{target} is the intersection of the image plane π_{image} with the plane through the camera center and parallel to π_{target} (cf., Hartley and Zisserman 2004, p. 217 and



Fig. 5. Computation of the vanishing line of a target plane rotated with respect to the image sensor plane

Fig. 5). Computing the vanishing line requires the normal vectors of π_{target} and π_{image} as well as the focal length *f* of the camera, i.e., the orthogonal distance of the camera center to π_{image} . The normal vector of the image plane is given by the theodolite angles

$$\mathbf{n}_{\text{image}} = \begin{bmatrix} \sin V \cos Hz \\ \sin V \sin Hz \\ \cos V \end{bmatrix}$$
(21)

and the focal length results from camera calibration, where $f \approx 105,000$ px for the used IATS (cf., the previous section).

By measuring a plurality of points ($N \ge 3$, not lying on a line) on the target plane, the normal vector of π_{target} results from a plane fit to these points. The points can either be measured by reflectorless polar measurements or, in this case, by using the IATS' scanning functionality. The plane fit to a large set of points can effectively be computed by the eigendecomposition (Shakarji 1998)

$$\mathbf{C} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^{I} \tag{22}$$

of the symmetric 3×3 covariance matrix

$$\mathbf{C} = [\mathbf{x} - \bar{x}, \, \mathbf{y} - \bar{y}, \, \mathbf{z} - \bar{z}]^T [\mathbf{x} - \bar{x}, \, \mathbf{y} - \bar{y}, \, \mathbf{z} - \bar{z}]$$
(23)

where \bar{x} , \bar{y} , and \bar{z} denote the mean values of the coordinate vectors **x**, **y**, and **z**. The normal vector of the target plane $\mathbf{n}_{\text{target}}$ is the eigenvector given by the column of **U** corresponding to the minimal eigenvalue given by the main diagonal of Λ .

After $\mathbf{n}_{\text{image}}$ and $\mathbf{n}_{\text{target}}$ are known in the same coordinate system, the vanishing line of the target plane can be computed by the intersection of the two planes. For evaluating Eq. (20), *d* can be computed as the orthogonal distance of the detected ellipse center to the vanishing line of π_{target} . The direction of the correction is given by the normal vector of the vanishing line \mathbf{n}_{VL} (with $|\mathbf{n}_{\text{VL}}| = 1$) in the image so that the corrected center coordinates finally read

$$\begin{bmatrix} u_{\rm corr} \\ v_{\rm corr} \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix} + \epsilon \mathbf{n}_{\rm VL}$$
(24)

where *u* and *v* result from Eq. (14); and the magnitude of the correction ϵ results from Eq. (20).

The effect of the ellipse center offset is demonstrated using data from measurements in the camera calibration field (cf., Fig. 11). The black circular targets [Fig. 6(a)] have a diameter of 40 mm and are used for the calibration of photogrammetric cameras. The targets also comprise a concentric white circle with a diameter of 2.5 mm.





Fig. 6. Effect of the ellipse center offset: (a) overview with large black circle and small white circle; (b) magnified detail with ellipse estimated from small white circle and ellipse center (cross) estimated from large black circle

Because of the elliptical shape of the imaged target in Fig. 6(a), it is obvious that $\pi_{image} \# \pi_{target}$ and one can expect an offset of the estimated ellipse center to the center of the actual target. According to Eq. (20), the center offset of the white ellipse with a small semiminor axis *b* is negligible ($\epsilon < 0.005$ px for Fig. 6), and its center corresponds to the center of the actual target. However, the center of the estimated ellipse based on the contour of the large black circle differs from the center of the small white ellipse [Fig. 6(b)].

For the demonstrated measurement configuration ϵ amounts to 1.3 px, which corresponds to 0.8 mgon for the used IATS. The consideration of the ellipse center offset is therefore inevitable for highly accurate measurements with large targets.

Fig. 7 shows the magnitude of the ellipse center correction ϵ for different target radii and angles β between the target plane and the image sensor plane. It is assumed that the ellipse center is detected in the principal point of the image sensor.

The relationship between the semiminor axis *b* of the mapped circle with a radius *R* is given by $b = R \cos \beta$ (Fig. 5). This explains the increasing value for ϵ with an increasing target radius because *R* occurs in the numerator of Eq. (25). The distance *d* of the ellipse



Fig. 7. Ellipse center corrections for different target radii and angles β between image sensor plane and target plane (ellipses with semiminor axes of less than 10 px are not considered): (a) maximum target radius of 250 px; (b) maximum target radius of 1,000 px

center, which is the principal point here, to the vanishing line is also given by $d = f/\tan\beta$ (Fig. 5) so that one can rewrite Eq. (20) to

$$\epsilon = \frac{(R\cos\beta)^2 \tan\beta}{f} \tag{25}$$

Setting the partial derivative

$$\frac{\partial \epsilon}{\partial \beta} = \frac{R^2}{f} \left(-2\cos\beta\sin\beta\tan\beta+\cos^2\beta\left(\tan^2\beta+1\right) \right)$$
$$= \frac{R^2}{f} \left(-2\sin^2\beta+1 \right) = 0$$
(26)

yields the maximum of the function. It is evident that $\partial \epsilon / \partial \beta = 0$ for sin $\beta = \sqrt{0.5}$, which is the case for $\beta = 50$ gon. This can also be observed in Fig. 7 in which the maximum corrections occur for $\beta = 50$ gon.

In Fig. 7 an image sensor with $2,560 \times 1,920$ px is assumed so that the maximum radius of the circle can be 1,920/2 px.



Fig. 8. Estimation of target coordinates based on the intersection of lines: (a) overview; (b) detail with lines found by the edge detector

Furthermore, resulting semiminor axes of less than 10 px are neglected. Considering a scale factor of about 0.61 mgon/px (cf., the previous section), Fig. 7 shows that the influence of the ellipse center offset is negligibly small (<0.1 mgon) for target radii <170 px. For larger targets, the offset must be considered because it can result in deviations of over 2 mgon. For the used IATS and a distance of, e.g., 10 m to the target, target radii of less than 16 mm result in imaged targets with radii of less than 170 px.

Edge Detector Offset

In Fig. 6(b) an offset of the edge detector generating the contour of the ellipse is visible. Because of the symmetry of the ellipse, the estimated center is not biased by the edge detector offset. However, for the computation of image coordinates based on the intersection of lines, this offset of the edge detector needs to be considered.

An example is the computation of the image coordinates of the markings of a tooling bar (Fig. 8). If only the lines l_1 and l_2 were used for the computation of the center coordinates by intersection, then the result would be biased by the offset of the edge detector. In the case of the marking depicted in Fig. 8, all lines are used to compute the center of the marking by a least-squares adjustment of the

lines' intersection. Because of the symmetry of the marking, the offset has no effect here.

Nevertheless, when using asymmetric targets, such as simple corners, the edge detector offset needs to be considered. This is especially important when defining the targets in the image by means of prominent image features because many feature detectors are based on corners (Tuytelaars and Mikolajczyk 2008, p. 216ff.). More information on the topic of the edge detector offset can be found in Gonzalez and Woods (2002, p. 572ff.) and Huertas and Medioni (1986).

Warm-Up Effects

Warm-up effects, i.e., the change of the measurement values caused by a change of the instrument temperature, are a well-known issue for IATS (Walser 2004, p. 25ff.; Wasmeier 2009, p. 73ff.; Knoblach 2011, p. 117ff.). The final measurement results of an IATS are composed of different individual measurements, such as theodolite angles and image-based measurements [cf., Eqs. (7) and (8)], but also tilt readings [cf., Eqs. (12) and (13)]. In this article, the warm-up effects of the system IATS are evaluated and the involved measurement sensors are not investigated individually.

For an experimental evaluation of the warm-up effects, the IATS was set up on a measurement pillar in the temperaturecontrolled laboratory with a basement decoupled from the building. There, the switched-off IATS was sufficiently (>12 h) acclimatized to the ambient temperature (20° C). After switching on the IATS, measurements to a circular target, mounted on another measurement pillar at a distance of 6.1 m, were conducted for over 3 h in which the telescope face was alternated after each measurement.

Fig. 9 depicts the variations of the measured angles and the internal temperature, which is measured on the mainboard of the IATS. The changes of the single-face measurements of up to 0.8 mgon are far beyond the measurement accuracy of about 0.1 mgon (cf., the next section) but are unsurprising because the instrument temperature increases from 20°C to over 30°C. However, for a stable setup of the IATS and the target one would expect no variations in the face averages of the angle measurements.

Therefore, the experiment was repeated, in which, the distance to the target, which was again mounted on a measurement pillar, was changed to 12.4 m. The variations of the face averages of the angle measurements show the same pattern as in Fig. 9 but are reduced by a factor 2. Because the distance to the target was roughly doubled with respect to the first experiment, it is concluded that the variations of the face averages result from small displacements of the IATS' alidade while warming up. This effect was also found by Wasmeier (2009, p. 77) for an IATS prototype.

In Fig. 10 the height change of the IATS, computed from the face average of the vertical angle in Fig. 9, is compared with the change of the instrument temperature. One observes that, after a dead time, the height of the IATS linearly increases with the instrument temperature. Apparently, the components of the IATS do not react to the temperature change as quickly as the temperature sensor, which causes the dead time in Fig. 10.

Total station-based measurements requiring high accuracies are generally performed in two telescope faces; therefore, the huge drifts of the single-face measurements in Fig. 9 are neglected in this article. However, the warming of the IATS also causes small displacements (a few 0.01 mm, cf. Fig. 10) of the alidade. For applications with the highest demands for accuracy (cf., the next section), a warm-up time of about 1 h should be followed to keep the





Fig. 9. Variations of horizontal and vertical (opposite sign for Face II) angle to a target at a distance of 6.1 m caused by a warming of the instrument



Fig. 10. Height change of the IATS caused by a warming of the instrument

remaining effects below 0.01 mm. It is emphasized that the experiments were performed with an instrument acclimatized to the ambient temperature. If there is a difference between storage and working temperature an additional warm-up time has to be considered.

Application: Geodetic Network Measurements

In the previous sections different error sources that can occur when working with an IATS were assessed. To evaluate the effect of the individual error sources, their impact on a typical measurement task, the determination of 3D coordinates by means of geodetic network measurements, was investigated. Under laboratory conditions and using an IATS prototype, Guillaume et al. (2012) established a small-scale geodetic network with empirical standard deviations of the 3D points of about 0.01 mm.

For the network measurements, the calibration field for photogrammetric cameras in the laboratory was chosen as a test bed (Fig. 11). The Points P1–P6 are circular targets with a diameter of 40 mm (Fig. 6), and T1 and T2 are the markings of an Invar tooling bar (Fig. 8). These eight points were observed from the Stations S1–S4 in which four sets of angles were measured at each station. To correct for the ellipse center offset, the target planes of the circular targets P1–P6 were measured by using the scanning functionality of the investigated IATS. To avoid influences caused by warmup effects (cf., the previous section), dummy measurements were performed for about 1.5 h before measuring the first set of angles.

For each observation in the Stations S1–S4, the center of the imaged target was within a distance of 150 px to the principal point of the image sensor. Errors caused by a simplified relationship of the image coordinates to theodolite angles or caused by an insufficient camera calibration are therefore expected to be negligible. To demonstrate the effect of these errors, Station SX was introduced. In Station SX also, four sets of angles were measured, but the targets P3, P4, and P6 were observed at larger distances to the principal point (Fig. 12). To evaluate the quality of the applied corrections, another station, SR, was introduced at the same position as SX but by observing all targets in the vicinity of the principal point.

Using the measurement data of the Stations S1–S4, SR, and SX, different 3D network adjustments were performed resulting in five test cases:

- References solution: Using Stations S1–S4 and SR and omitting SX, using a thorough relationship between image coordinates and theodolite angles according to Eqs. (7) and (8), application of the camera calibration parameters for the actual focus position, correction of the detected ellipse center according to Eq. (24), and tilt correction according to Eqs. (12) and (13)
- Thorough modeling: Like Case 1 but using Station SX and omitting SR
- 3. Simplified relationship: Like Case 2 but using Eqs. (9) and (10) instead of Eqs. (7) and (8) for relating the image coordinates to theodolite angles
- Simple calibration: Like Case 2 but omitting rotation of the image sensor κ and distortion K₁ and using an average focal length (f = 104,560 px) for all focus positions
- 5. Raw ellipse center: Like Case 2 but omitting ellipse center correction

The evaluation of the measured sets of angles resulted in empirical standard deviations for the horizontal and vertical angle between 0.02 and 0.09 mgon. These results are invariant to the applied corrections because the errors discussed in the previous sections are deterministic and thus cannot be detected by repeated measurements, such as a set of angles.



Fig. 11. Measurement site with Stations S1–S4, SR, and SX and Targets P1–P6, T1, and T2

In the adjustment of the network the scale was fixed by the distance between the markings of the Invar tooling bar $(1.308129 \text{ m} \pm 2 \ \mu \text{m})$, and the Points P1–P6 were chosen to define the datum. The initial standard deviation of the angle measurements (horizontal and vertical) was set to 0.1 mgon. After the network adjustment, the empirical standard deviations of the horizontal and vertical angle were determined by a variance component estimation. The geodetic network was then readjusted using these empirical standard deviations.

Table 2 summarizes the estimated standard deviations for horizontal s_{Hz} and vertical s_V angle measurements resulting from the variance component estimation for the five test cases. The estimated standard deviations of about 0.1 mgon gained from a thorough modeling of the discussed error sources (Test Case 2) are in correspondence to the reference solution (Test Case 1). This standard deviation for image-based measurements was also found by Ehrhart and Lienhart (2015a) for the predecessor (Leica MS50 I R2000) of the IATS used in this work. A similar value of 0.15 mgon is also reported by Guillaume et al. (2012) for an IATS prototype. The reason for the higher standard deviation for vertical angle measurements is the setting accuracy of the IATS' compensator of 0.15 mgon (Leica 2015, p. 80) which, according to Eq. (13), affects vertical angle measurements directly. A possible strategy to bypass this limitation is to disable the compensator during the measurements. The tilt of the IATS in each station can then be estimated by using a bundle adjustment (Kraus 2007, p. 269ff.) in the computations of the network adjustment.

Using a simplified relationship between image coordinates and theodolite angles (Test Case 3) or a simple camera calibration (Test Case 4) significantly (confidence level of 95%) degrades the quality of the results (Table 2). It is again emphasized that the standard

deviations resulting from the measured sets of angles (values between 0.02 and 0.09 mgon for all test cases) are no reliable estimates for the measurement accuracy because the discussed error sources are deterministic and cannot be detected by repeated measurements.

As shown in Fig. 6, the ellipse center offset can cause errors in the range of 0.8 mgon for selected measurements in the geodetic network. Therefore, one would expect to see these errors in the results of Test Case 5. However, the estimated standard deviations resulting from an omission of the ellipse center offset (Test Case 5) show nonsignificant differences to the reference solution (Table 2). This can be explained by the geometry of the network (Fig. 11). All stations are approximately at the same height and are somewhat symmetrically distributed. The observations from, e.g., S1 to P3 and S2 to P4 are equally affected by the ellipse center offset (the respective points form a parallelogram). Similar constellations (e.g., S1 to P4 and S2 to P5) can be found throughout the network. This leads to systematic errors that cannot be detected for the given geometry of the network.

Table 3 lists the empirical standard deviations of the estimated coordinates resulting from the network adjustments of the five test cases. Only the results for the target points P1–P6 are shown because the coordinates of the stations and the markings of the tooling bar have no representation after the IATS or the tooling bar is removed from its position.

As for the estimated standard deviations of the angle measurements (Table 2), the standard deviations of the coordinates gained from a thorough modeling of the discussed error sources (Test Case 2) correspond to the reference solution (Test Case 1). Along with the measurement precision, the standard deviations of the estimated



Fig. 12. Locations of targets observed from Station SX on the image sensor

Table 2. Standard Deviations of the Angle Measurements Resulting from

 Variance Component Estimation for Different Test Cases

Test case	s _{Hz} (mgon)	$s_{\rm V}$ (mgon)
1	0.06	0.11
2	0.06	0.11
3	0.12	0.11
4	0.85	0.33
5	0.08	0.10

Note: Significant (95%) deviations to reference (Test Case 1) are shown in boldface.

coordinates are a function of the geometry of the geodetic network. This explains why the standard deviations in vertical direction s_Z tend to be smaller than the standard deviations in horizontal direction s_X and s_Y (Table 3), although s_{Hz} is superior to s_V for Test Cases 1 and 2 (Table 2). It is further emphasized that the shown values can be outperformed for more favorable network geometries (Guillaume et al. 2012).

The standard deviations of the coordinates (Table 3) gained from a simplified relationship between image coordinates and theodolite angles (Test Case 3) or a simple camera calibration (Test Case 4) significantly (confidence level of 95%) differ from the reference solution (Test Case 1). Compared with the reference solution, the standard deviations in horizontal direction s_X and s_Y roughly increase proportionally to the increase of s_{Hz} in Table 2. The standard deviations in vertical direction s_Z of Test Case 3 significantly differ from the reference solution (Test Case 1), although the respective $s_{\rm V}$ values are identical (Table 2). This is explained by the alignment of the Invar tooling bar (Fig. 11), which is used to determine the scale of the geodetic network. Because the tooling bar is aligned horizontally, horizontal angle measurements are needed to determine the scale. Consequently, an increasing standard deviation of the horizontal angle measurement (compare Test Cases 1 and 3 in Table 2) increases the standard deviation of the scale, which then increases the standard deviation in vertical direction s_{Z} .

As for the estimated standard deviations of the angle measurements (Table 2), the standard deviations of the coordinates (Table 3) gained from an omission of the ellipse center offset (Test Case 5) show nonsignificant differences to the reference solution (Test Case 1). The explanation is analog to the discussion of Test Case 5 in Table 2, in which the regular geometry of

Table 3. Empirical Standard Deviations of the Estimated Coordinates for

 Different Test Cases

Point	Test case	$s_X(\mathrm{mm})$	$s_Y(mm)$	$s_Z (\mathrm{mm})$
P1	1	0.021	0.018	0.005
	2	0.021	0.018	0.005
	3	0.041	0.034	0.009
	4	0.273	0.224	0.060
	5	0.028	0.023	0.007
Р2	1	0.013	0.017	0.006
	2	0.013	0.017	0.006
	3	0.026	0.033	0.008
	4	0.175	0.222	0.050
	5	0.018	0.023	0.006
P3	1	0.009	0.015	0.007
	2	0.009	0.015	0.007
	3	0.016	0.029	0.011
	4	0.090	0.198	0.064
	5	0.011	0.020	0.008
P4	1	0.008	0.007	0.006
	2	0.008	0.007	0.006
	3	0.015	0.011	0.010
	4	0.084	0.060	0.060
	5	0.011	0.008	0.007
Р5	1	0.008	0.021	0.007
	2	0.008	0.021	0.007
	3	0.015	0.042	0.011
	4	0.097	0.285	0.064
	5	0.011	0.028	0.008
P6	1	0.009	0.022	0.006
	2	0.009	0.022	0.006
	3	0.017	0.042	0.010
	4	0.110	0.290	0.061
	5	0.011	0.029	0.007

Note: Significant (95%) deviations to reference (Test Case 1) are shown in boldface.

the network was found to be inadequate to detect errors caused by the ellipse center offset.

Therefore, the deviations of the coordinates with respect to the reference solution (Test Case 1) for the other four test cases were investigated (Table 4). Again, the results from a thorough modeling of the errors (Test Case 2) have the lowest deviations, and the results from a simple camera calibration (Test Case 4) have the highest deviations. The coordinate deviations resulting from a simplified relationship between image coordinates and theodolite angles (Test Case 3) are small because only three measurements were taken at unfavorable positions on the image sensor (Fig. 12). With deviations of up to 0.07 mm for Test Case 5, it is demonstrated that the ellipse center offset must be taken into account for highly accurate measurements to large circular targets.

Conclusions

In this paper, different error sources that limit the accuracy of IATS measurements were investigated. The modeling of the relationship between image coordinates and theodolite angles, the calibration of the camera, and target-specific problems were addressed. The impact of the different error sources for varying constellations, such as the position of the target on the image sensor and the vertical angle of the telescope, were studied. For the

 Table 4. Coordinate Deviations to the Reference Solution for Different Test Cases

Point	Test case	$\Delta X (\mathrm{mm})$	$\Delta Y (\mathrm{mm})$	$\Delta Z (\mathrm{mm})$
P1	2	-0.003	-0.005	0.000
	3	-0.002	0.000	0.002
	4	-0.108	-0.152	-0.017
	5	-0.024	-0.013	-0.018
P2	2	-0.001	0.000	0.001
	3	0.006	-0.003	0.003
	4	-0.014	-0.020	-0.004
	5	0.010	-0.050	-0.011
Р3	2	0.000	0.001	-0.003
	3	-0.004	-0.004	-0.004
	4	-0.009	0.033	-0.026
	5	0.016	-0.063	0.011
P4	2	0.001	0.001	0.000
	3	0.002	0.003	-0.005
	4	0.077	0.029	0.037
	5	-0.005	-0.003	0.018
P5	2	0.003	0.004	0.001
	3	-0.001	0.003	0.001
	4	0.053	0.109	0.009
	5	0.003	0.064	0.015
Р6	2	0.000	-0.001	0.001
	3	-0.001	0.000	0.003
	4	0.002	0.000	-0.001
	5	0.000	0.065	-0.016

investigated IATS, it was demonstrated that warm-up effects can be avoided to a great extent when performing two-face measurements, and a warm-up time of about 1 h was identified for applications with highest demands for accuracy (0.01 mm).

It was pointed out that errors in the camera calibration and errors caused by a simplified relationship between image coordinates and theodolite angles can be avoided when observing targets that are imaged in the vicinity of the principal point of the image sensor. For circular targets, it was shown that the center of the imaged ellipse does not correspond to the center of the actual target. The offset can either be modeled with knowledge of the relative orientation between the image sensor plane and the target plane or can be kept small by using targets with small diameters. It was further demonstrated that errors caused by edge detector offsets can be avoided by using symmetric visual targets.

In experimental measurements under laboratory conditions an accuracy of 0.1 mgon for image-based angle measurements was identified with a commercially available IATS. It was shown that the results of repeated measurements, such as sets of angles, do not deliver reliable estimates for the measurement accuracy because they do not account for systematic error sources. It is rather necessary to estimate the measurement accuracy from overdetermined measurement configurations, such as a geodetic network.

With the investigated IATS it is possible to perform automated, highly accurate angle measurements with simple visual targets. For conventional total stations, this is only possible with retroreflective prisms. Along with saving of costs, visual targets are also beneficial because they allow a direct measurement of the signalized point. It is therefore possible to measure the same points with tactile measurement sensors. This is not possible for points signalized by retroreflective prisms because their coordinates refer to the inaccessible prism center. In cases in which the object to be measured contains favorable visual structures by itself, attaching additional visual markers is not necessary, and the IATS provides a highly accurate and fully contactless measurement system.

References

- Ahn, S. J., Warnecke, H. J., and Kotowski, R. (1999). "Systematic geometric image measurement errors of circular object targets: Mathematical formulation and correction." *Photogramm. Rec.*, 16(93), 485–502.
- Bürki, B., Guillaume, S., Sorber, P., and Oesch, H. P. (2010). "DAEDALUS: A versatile usable digital clip-on measuring system for total stations." *Proc., Int. Conf. on Indoor Positioning and Indoor Navigation (IPIN)*, IEEE, New York, 1–10.
- Davies, E. R. (2012). Computer and machine vision, 2nd Ed., Academic Press, Boston.
- Dold, J. (1996). "Influence of large targets on the results of photogrammetric bundle adjustment." *International Archives of Photogrammetry and Remote Sensing, XXXI*, International Society for Photogrammetry and Remote Sensing, Hannover, Germany, 119–123.
- Ehrhart, M., and Lienhart, W. (2015a). "Image-based dynamic deformation monitoring of civil engineering structures from long ranges." *Proc.*, *SPIE 9405, Image Processing: Machine Vision Applications VIII*, SPIE, Bellingham, WA, 94050J.
- Ehrhart, M., and Lienhart, W. (2015b). "Monitoring of civil engineering structures using a state-of-the-art image assisted total station." *J. Appl. Geod.*, 9(3), 174–182.
- Fitzgibbon, A. W., Pilu, M., and Fisher, R. B. (1996). "Direct least squares fitting of ellipses." *Proc.*, *13th Int. Conf. on Pattern Recognition*, Vol. 1, IEEE Computer Society, Washington, DC, 253–257.
- Gonzalez, R. C., and Woods, R. E. (2002). Digital image processing, 2nd Ed., Prentice Hall, Upper Saddle River, NJ.
- Guillaume, S., Bürki, B., Griffet, S., and Mainaud Durand, H. (2012). "QDaedalus: Augmentation of total stations by CCD sensor for automated contactless high-precision metrology." *FIG Working Week 2012, TS09I–Engineering Surveying*, FIG, Copenhagen, Denmark, 1–15.
- Hartley, R., and Zisserman, A. (2004). Multiple view geometry in computer vision, 2nd Ed., Cambridge University Press, Cambridge, U.K.
- Hauth, S., Schlüter, M., and Thiery, F. (2013). "Schneller und ausdauernder als das menschliche Auge: Modulare Okularkameras am Motortachymeter." *Allg. Verm.-Nachr.*, 120(6), 210–216 (in German).
- Huang, Y. D., and Harley, I. (1989). "Calibration of close-range photogrammetric stations using a free network bundle adjustment." *Proc., Optical 3-D Measurement Techniques*, Herbert Wichmann Verlag GmbH, Karlsruhe, Germany, 49–56.
- Huertas, A., and Medioni, G. (1986). "Detection of intensity changes with subpixel accuracy using Laplacian-Gaussian masks." *IEEE Trans. Pattern Anal. Mach. Intell.*, 8(5), 651–664.
- Juretzko M., (2004). "Reflektorlose Video-Tachymetrie–ein integrales Verfahren zur Erfassung geometrischer und visueller Informationen." Ph.D. thesis, Ruhr-Universität Bochum, Bochum, Germany, 122 (in German).
- Knoblach, S. (2011). "Entwicklung, Kalibrierung und Erprobung eines kameraunterstützten Hängetachymeters." Ph.D. thesis, DGK C-655, Technische Univ. Dresden, Dresden, Germany, 146 (in German).
- Kraus, K. (2007). Photogrammetry: Geometry from images and laser scans, 2nd Ed., Walter de Gruyter, Berlin.
- Leica. (2015). Leica MS60/TS60 user manual, Version 1.0, Leica Geosystems AG, Heerbrugg, Switzerland, 90.
- Luhmann, T. (2014). "Eccentricity in images of circular and spherical targets and its impact on spatial intersection." *Photogramm. Rec.*, 29(148), 417–433.
- Reiterer, A., Huber, N. B., and Bauer, A. (2010). "Image-based point detection and matching in a geo-monitoring system." *Allg. Verm.-Nachr.*, 117(4), 129–139.
- Scherer, M. (2007). "Phototachymetrie: Eine Methode zur Bauaufnahme und zur Erstellung eines virtuellen Modells." *Allg. Verm.-Nachr.*, 114(8–9), 307–313 (in German).

- Schirmer, W. (1994). "Universaltheodolit und CCD-Kamera–ein unpersönliches Meßsystem für astronomisch-geodätische Beobachtungen." Ph.D. thesis, DGK C-427, Technische Univ. München, Munich, Germany, 93 (in German).
- Shakarji, C. M. (1998). "Least-squares fitting algorithms of the NIST algorithm testing system." J. Res. Nat. Inst. Stand. Technol., 103(6), 633–641.
- Snyder, J. P. (1987). "Map projections—A working manual." Paper 1395, U. S. Geological Survey, U.S. Government Printing Office, Washington, DC.
- Topcon. (2016). "IS-3 imaging station brochure." Topcon Corporation, Livermore, CA, 4.
- Tuytelaars, T., and Mikolajczyk, K. (2008). "Local invariant feature detectors: A survey." *Found. Trends Comput. Graphics Vision*, 3(3), 177–280.

- Wagner, A., Wasmeier, P., Reith, C., and Wunderlich, T. (2013). "Bridge monitoring by means of video-tacheometer–A case study." *Allg. Verm.-Nachr.*, 120(8–9), 283–292.
- Wagner, A., Wasmeier, P., Wunderlich, T., and Ingensand, H. (2014). "Vom selbstzielenden Theodolit zur Image Assisted Total Station." *Allg. Verm.-Nachr.*, 121(5), 171–180 (in German).
- Walser, B. (2004). "Development and calibration of an image assisted total station." Ph.D. thesis, Blaue Reihe 87, Swiss Federal Institute of Technology, Zurich, Switzerland, 168.
- Wasmeier, P. (2009). "Grundlagen der Deformationsbestimmung mit Messdaten bildgebender Tachymeter." Ph.D. thesis, DGK C-638, Technische Univ. München, Munich, Germany, 149 (in German).